

3.11 Sum and Difference Identities for Sine and Tangent

There are also sum and difference identities for sine and tangent:

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

Mnemonic for sine sum/difference: "Sine can't change signs."

Examples:

Find the exact value of $\sin(75^\circ)$.

Find the exact value of $\sin(\pi/12)$.

Find the exact value of $\tan(15^\circ)$.

Find the exact value of $\tan(5\pi/12)$.

Find the exact value of $\sin(\alpha + \beta)$ if $\sin \alpha = -3/5$ and $\cos \beta = -1/3$ with α in Quadrant IV and β in Quadrant III.